The Optimal Allocation of the High School Senior Year:

Solving for the distribution of time across Standardized Tests and Grade Point Average

.

Abstract

College-bound high school students are faced with an unparalleled and worsening situation: the admissions process is increasingly daunting as population and applicants are increasing more rapidly than seats in universities.[1]

Too often students plan their time as if it is unlimited. This is a byproduct of excessive homework, extracurricular demands, standardized test prep, and personal demands, all which demand attention simultaneously.

A more economic approach is needed. Time is not unlimited, the admissions process is well-understood, GPAs are calculable, and point-gains from standardized-test prep are predictable. There is a way to solve for the optimal time allocation of a high school senior: to specifically elucidate how a student should spend his last 3 months prior to applications to maximize his odds of admissions.

Background

Entering their senior year of high school, students are in the position of having to allocate time between classes, standardized tests, and extra-curricular activities—the three factors that colleges regard most when considering a student for admission.

While it is true that a student must always—throughout high school—be managing time between these three areas, the senior year is unique for two reasons. First, most students will be taking the most rigorous coursework of their high school careers. Second, this is a student's last chance to take the SAT or ACT. During a student's senior year, there are three sittings of each test prior to the admissions deadlines imposed by most four-year universities. Hence the student is in the unique position of having to allocate his time between the most rigorous coursework of his thus-far career, and preparing for the SAT and ACT.

One may argue that students would do well to take their standardized test earlier, but two things make this imprudent. First, scores tend to increase as a student matures in his education[2], so senior year is the optimal time to test. Second, the summer leading up to the fall of the senior year provides ample time to prepare for the test. Junior year does not afford the same prep time as one has a full course load.

A student then finds himself in his senior year, educationally at his best, and with a summer's worth of prep behind him, facing down the most challenging curriculum of his career. The optimal allocation of a student's time in the months leading up to the October, November, and December tests is the concern of this analysis.

---

[1] http://www.theatlantic.com/education/archive/2014/04/is-college-really-harder-to-get-into-than-it-used-to-be/360114/

[2] http://sat.collegeboard.org/register/when-to-take-sat

Analysis

Assumptions, and Quantifying Time

An analysis comparing time spent on one activity versus another requires fixed units of time to make the comparison equitable. In most cases, this is simple: use hours. However, in this analysis, that would be myopic, as it takes some students twice as long to achieve the same results as others.

A better way to conceptualize time, for the purposes of this analysis, is in units. We will allow that a student has 7 units of time per semester. This can best be conceptualized as, 7 units of time to do all his homework, prepare for class, and study. 7 was chosen because it fits nicely with the 7 classes per semester that we will assume each student takes, but neither of these assumptions is important: the math and the results all hold true with any number of units of "time" and any number of classes. The only thing that matters is that those 7 units are fixed, and cannot be stretched. That is, an all-nighter would not increase the number of units of time, they would be encapsulated in the 7-units assumption.

To break this down in terms of their impact on GPA and SAT scores:

In Terms of SAT		In Terms of Grade Point Average	
Units of Time Per Semester (AKA classes)	7.00	Units of Time Per Semester (AKA classes)	7.00
SAT Gains Per Week	20.00	Number of Classes	7.00
Weeks per Semester	18.00	Number of Potential Grade Points per Class	4.00
Potential SAT Points	360.00	Total Potential Grade Points	28.00
SAT Points per Unit of Time	51.43	Grade Points per Unit of Time	4.00
Cost of 1 SAT Point (In Units of Time)	0.02	Cost of 1 Grade Point (In Terms of Units of Time)	0.25
Cost of 360 Points	7.00	Cost of an A	1.00
Cost of 300 Points	5.83	Cost of a B	0.75
Cost of 200 Points	3.89	Cost of a C	0.50
Cost of 100 Points	1.94	Cost of D	0.25
Cost of 0 Points	0.00	Cost of an F	0.00

Other relevant assumptions include the premises:

- That a student can gain 20 points on the SAT each week. This is firmly rooted in a statistically significant sampling of Powerful Prep's students.
- That a student has taken 7 courses each semester of school.
- That grade points are distributed discretely: 1,2,3,4; not 1.23234, 2.2234
- That the average GPA is 3.0[3], and the standard deviation of GPA is 0.5, according to US news and world, and *Statistics* by Freedman, Pisani, Purves et al. [4]
- That the time and effort required to generate 7-classes worth of straight-As, is the same as that required to generate a 360-point gain on the SAT—this is an extraordinarily conservative assumption, as empirically, we know that it is quite common to generate 360 point gains *and* a 4.0 GPA. More likely, it requires far less than 7 units of time to generate 360 points of SAT gains.

[3] http://www.usnews.com/opinion/articles/2011/04/19/average-high-school-gpas-increased-since-1990
[4] *Statistics* by Freedman, Pisani, Purves et al

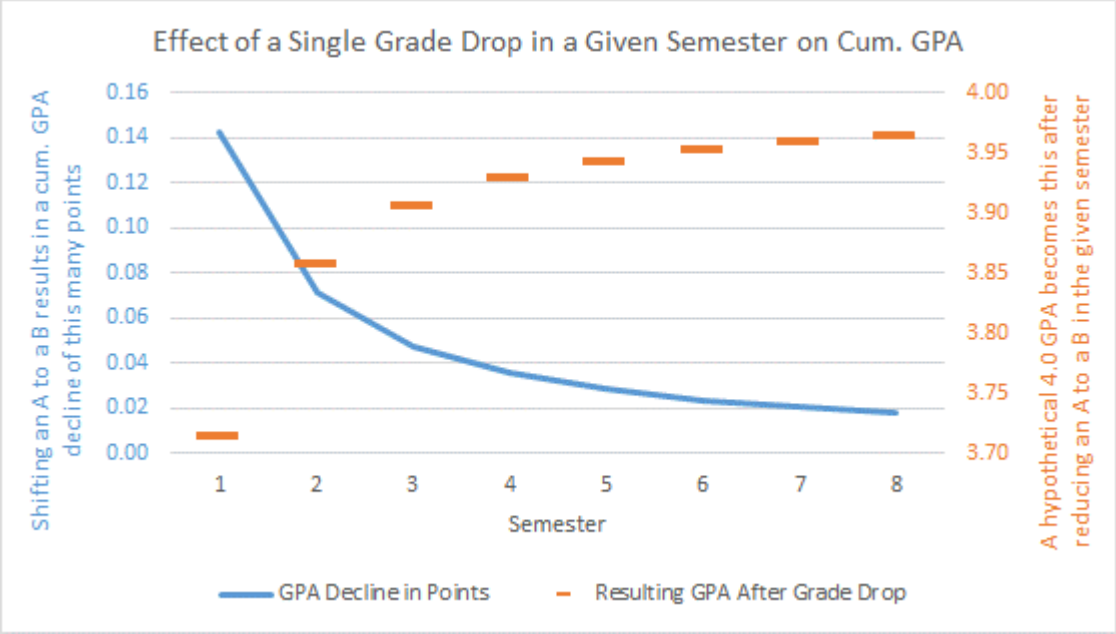
The Fixed Nature of Grade-Point Averages

The Grade-Point Average (GPA) is the average of the point value of each grade in a student's transcript. A grade of an A is equivalent to 4 points, a B is 3, a C is 2, a D is 1, and an F is 0.

While there are distinctions between Weighted and Unweighted GPAs—Weighted GPAs grant an extra point in Advanced Placement Classes—Weighted GPAs are unworthy of consideration here, as colleges regard the unweighted GPA in their analysis.

The GPA is, above all and often-forgotten, an average. Averages have the unique property of becoming intransigent as more points are added to them. Thus, a hypothetical 3.0 GPA is more solid from a Junior than it is from a Sophomore, and a deviation from the average will hold less sway on the GPA of a Junior than a Sophomore.

This fact and nature is quantifiable.



This graph illustrates the point: each subsequent semester, the effect of a single grade decline has lower impact on the overall GPA. In the 8th semester, a 4.0 GPA moves to 3.98: only 0.02 Grade Points are shaved off the average by dropping a single class's grade down one letter grade.

The 7th semester is most applicable, this is the first semester of senior year. We see that a student loses only slightly more than 0.02 points off his average by losing a full letter grade in one class. This effect holds for all GPAs and works in the opposite as well: moving a grade *up* in a class has a similarly negligible effect in the 7th semester. That is moving from a B to an A in a class only moves a GPA *up* ~0.02 points.

The effect holds for all GPAs: each letter grade drop in the 7th semester has only a 0.021 effect on the overall GPA, regardless of GPA. Thus a 4.0 becomes a 3.98 (a drop of 0.024) when the student replaces an average grade (a 4.0 in this case) with a one-point lower grade (a 3.0 grade). Similarly, a 3.0 becomes a 2.98 (a drop of 0.024) with the replacement of a 3-point grade with a 2-point grade (e.g. a B is replaced by a C).

Overall GPA Response to a Single B: Assumes GPA Starts at 4.0 and 7 Classes per Semester

Semester	GPA Post Grade Drop	Sensitivity to grade drop
1	3.71	0.143
2	3.86	0.071
3	3.90	0.048
4	3.93	0.036
5	3.94	0.029
6	3.95	0.024
7	3.96	0.020
8	3.96	0.018

Overall GPA Response to a Single C: Assumes GPA Starts at 3.0 and 7 Classes per Semester

Semester	GPA Post Grade Drop	Sensitivity to grade drop
1	2.86	0.143
2	2.93	0.071
3	2.95	0.048
4	2.96	0.036
5	2.97	0.029
6	2.98	0.024
7	2.98	0.020
8	2.98	0.018

In percentile terms, the change in GPA is -1.5% in the shift from 4.0 to 3.98. The percentile shift from 3.0 to 2.98 is -1.5% as well. Note that it is not 0.79% (2.98/3.00-1 = 0.0079) because the progression of GPA percentiles is non-linear and is calculated using the statistical z-scores of the starting and ending GPAs.

A senior then finds himself spending 1 unit of time trying to achieve a return of 1.5% (the GPA improvement in keeping a perfect 4.0 from slipping to 3.98). Is this an appropriate use of time? Ideally, a student would not spend more effort than reward. Consider both in terms of percentages to see the effect: 1 unit of time is 2% of a high-school senior's total available time (1 unit / (7 units per semester x 7 semesters)). Then a senior is getting a return of 1.5% for an effort of 2.0%. This is a negative return. To see this on a wider scale, consider the impact of receiving all of one grade (straight-As; straight Bs, etc.)

		GPA After Addition of 7 of This Grade			Starting Percentile		
Grade	Effort Required To Achieve	Starting with 4.0	Starting with 3.5	Starting with 3.0	Starting with 4.0	Starting with 3.5	Starting with 3.0
A	14.29%	4.00	3.57	3.14	99%	85%	51%
B	10.71%	3.86	3.43	3.00	99%	85%	51%
C	7.14%	3.71	3.29	2.86	99%	85%	51%
D	3.57%	3.57	3.14	2.71	99%	85%	51%
F	0.00%	3.43	3.00	2.57	99%	85%	51%
		Ending Percentile			Return on Time		
Grade	Effort Required To Achieve	Starting with 4.0	Starting with 3.5	Starting with 3.0	Starting with 4.0	Starting with 3.5	Starting with 3.0
A	14.29%	99%	88%	62%	0%	26%	154%
B	10.71%	97%	81%	51%	-19%	-41%	0%
C	7.14%	93%	73%	40%	-76%	-206%	-309%
D	3.57%	88%	62%	29%	-294%	-753%	-1187%
F	0.00%	81%	51%	21%	N/A	N/A	N/A

From this, one can see that as a 7th-semester senior beginning with a 4.0 has no way to generate a positive return on time. In fact, the scenario where any student can generate a positive return on time is rare. One would need to enter senior year with a 3.0 and have a straight-A semester to generate a positive return on the time he invested.

The critic would suggest that even though the situation may appear dire, all are in the same boat, and there is no way around it: a student cannot elect to stop schooling, so he is stuck in this losing battle. In the GPA realm, this is true, time invested generally yields negative returns. In the standardized test realm, this is not so.

The Flexible Nature of Standardized Test Scores

Standardized test scores are, by nature, very mobile. SAT scores and ACT scores vary from test-to-test, and schools usually will allow a student's highest score to serve as his representation. Some schools even super-score, allowing a student to use his best single-section score from multiple tests to serve as his representative score (e.g. 700 in math from an October test plus 650 in Reading from a November test plus 690 in Writing from a December test = 2040). This variability in test scores is in stark contrast to the GPA which becomes more and more fixed as time passes.

It is somewhat difficult to approximate the time that a student will need to prepare for his standardized tests, but the exact hours are not relevant to this analysis. We can use our already-established construct of class-units. Consider that a student treats test-prep as a class, and forgoes his class-work to prepare for the SAT. His return on time-spent is modeled as follows:

Liberal Time Estimate

Liberal Time Estimate		Percentage-Point Gain			Return on Effort		
Point Gains	Effort Required To Achieve	Starting with 2000	Starting with 1750	Starting with 1500	Starting with 2000	Starting with 1750	Starting with 1500
360	2.86%	6.9%	19.0%	35.0%	142%	565%	1125%
300	2.14%	6.9%	17.0%	31.0%	222%	693%	1347%
200	1.43%	5.0%	13.0%	22.0%	250%	810%	1440%
100	0.71%	3.0%	7.0%	12.0%	320%	880%	1580%
0	0.00%	0.0%	0.0%	0.0%	N/A	N/A	N/A

Conservative Time Estimate

Conservative Time Estimate		Percentage-Point Gain			Return on Effort		
Point Gains	Effort Required To Achieve	Starting with 2000	Starting with 1750	Starting with 1500	Starting with 2000	Starting with 1750	Starting with 1500
360	17.14%	6.9%	19.0%	35.0%	-60%	11%	104%
300	12.86%	6.9%	17.0%	31.0%	-46%	32%	141%
200	8.57%	5.0%	13.0%	22.0%	-42%	52%	157%
100	4.29%	3.0%	7.0%	12.0%	-30%	63%	180%
0	0.00%	0.0%	0.0%	0.0%	N/A	N/A	N/A

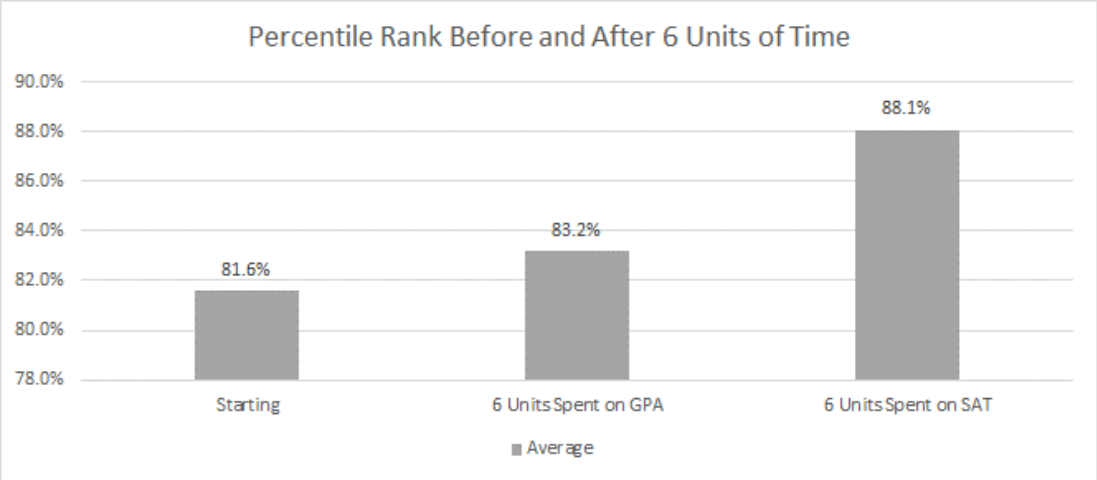
In contrast to the mostly-negative returns of the GPA efforts of a 7th-semester senior, the returns on time-investments for test-prep are skewed positive, and yield as much as 1580 percent in this analysis.

Even a meager gain of 50 points (answering an additional 5 out of 170 questions correctly) has a positive return-on-time-invested. Clearly, if colleges equally weight standardized test scores and grade-point averages, then a senior should focus far more time on the former than the later. In fact, colleges would have to massively overvalue GPA relative to standardized test scores in order for it to make sense to allocate more time to GPA improvement than standardized test score improvement.

The Profile of an Applicant (3 Factors)

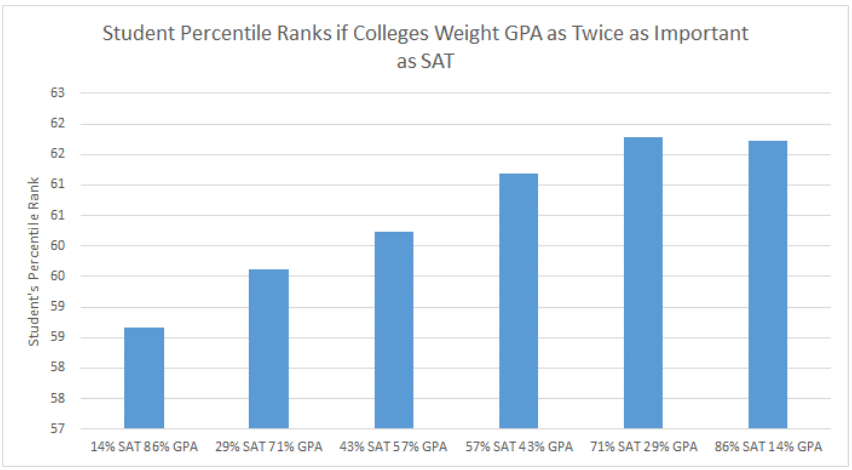
It is impossible to perfectly solve for the optimal time-allocation of time for a 7th-semester senior because it is impossible to know exactly how admissions offices weight GPA, standardized-test scores, extra-curricular achievements, and minority status. However, by evaluating the return on time-spent under different hypothetical values, the bounds can be seen.

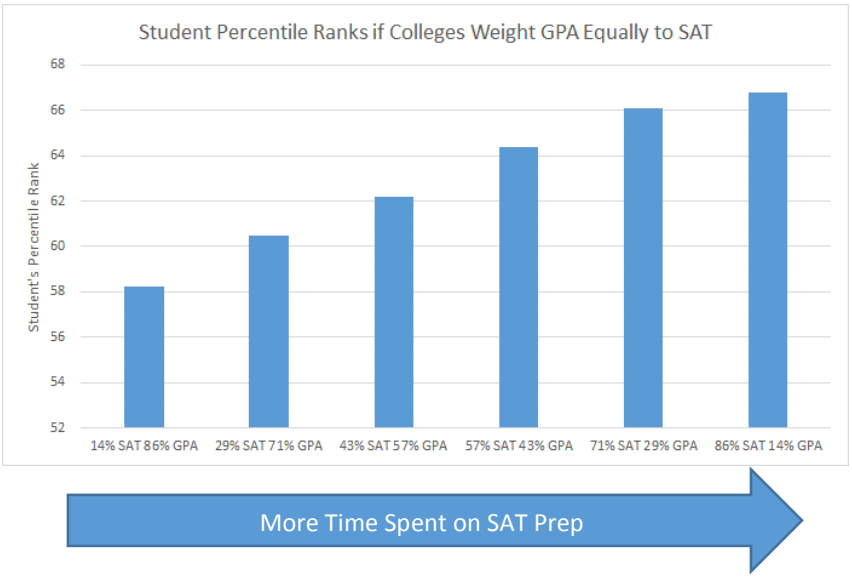
First, assume a simple case: that GPA and test scores are viewed equally by college admissions officers, then analyze the effects of 6 units of time spent on a student beginning with a 3.5 GPA and a 1750 SAT score.

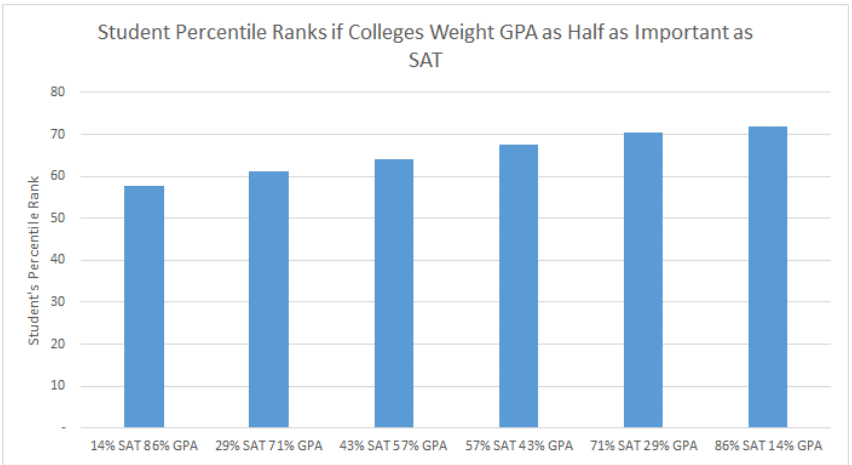


Rank	Starting	6 Units Spent on GPA	6 Units Spent on SAT
SAT Percentile	78.0%	78.0%	91.0%
GPA Percentile	85.1%	88.3%	85.1%
Average	**81.6%**	**83.2%**	**88.1%**

After 6 units of time spent, a student can improve from 78th percentile to 91st percentile in the SAT—a gain of 13 percentile points. Spending the same amount of time on ones 7th semester GPA would only yield 3 percentile points of improvement. Looking at the SAT and GPA percentiles combined gives a general estimate of the percentile rank of the candidate. Clearly, spending more time on the SAT raises the candidate's percentile the highest. The assumption here is that the candidate's percentile is an average of his SAT percentile and his GPA percentile. However, in truth, different schools will weight standardized test scores and GPAs differently. To see the effects of this, we can apply different weights to create the average.


Student Percentile Ranks if Colleges Weight GPA as Twice as Important as SAT
Student's Percentile Rank
63
62
62
61
61
60
60
59
59
58
58
57
14% SAT 86% GPA
29% SAT 71% GPA
43% SAT 57% GPA
57% SAT 43% GPA
71% SAT 29% GPA
86% SAT 14% GPA
Higher Percentile Rank
More Time Spent on SAT Prep


Student Percentile Ranks if Colleges Weight GPA Equally to SAT
Student's Percentile Rank
68
66
64
62
60
58
56
54
52
14% SAT 86% GPA
29% SAT 71% GPA
43% SAT 57% GPA
57% SAT 43% GPA
71% SAT 29% GPA
86% SAT 14% GPA
More Time Spent on SAT Prep


Student Percentile Ranks if Colleges Weight GPA as Half as Important as SAT
Student's Percentile Rank
80
70
60
50
40
30
20
10
-
14% SAT 86% GPA
29% SAT 71% GPA
43% SAT 57% GPA
57% SAT 43% GPA
71% SAT 29% GPA
86% SAT 14% GPA

These charts illustrate a student's overall rank by combining the SAT and GPA with different weightings. The hypothetical student in this graph has a 3.0 GPA and a 1500 SAT starting score.

Each graph illustrates the same effect: as a student spends more time preparing for the SAT, he becomes a higher-percentile-rank candidate, even independently of how the schools view the SAT in relation to GPA. Even a student pursuing a school that treats GPA with twice the weight of standardized test scores will benefit from overallocating time to SAT prep.

Recommendation

This analysis was catalyzed by years of seeing high school seniors subordinate SAT and ACT prep to school work. Anyone familiar with motivated high school seniors will witness students pour their time into their school-assigned homework, trying to achieve As that will move their GPAs barely 0.02 points. Worse still is that students do not seem to realize the poverty of the return they are fighting so hard for. A very few students can answer the questions "how much will your GPA improve by if you get strait As this semester?" "How much will it decline by if you get straight Bs?" They simply go through the motions of their classmates, and burn the midnight oil trying to make the grade.

That attitude is commendable if a student's sole goal is to become valedictorian—an accolade based on GPA only. But this is not most students' goal. Instead, most are seeking to gain admissions to colleges, a process which requires both a strong GPA and strong standardized test scores. And, frustratingly, students are taught to sink time into the endeavor with the least return, and to allot time to test prep—the highest return—if there is any left over.

The assumptions and analysis in this paper assume that a student can accurately manipulate his GPA very precisely, which, in some cases is not a feasible assumption. To implement the recommendations of this analysis, with the understandable constraint that students would be very hard-pressed to perfectly manipulate their GPA, there are a multiple techniques to employ.

First, a student can prioritize his work: complete test-prep before school work. In this, a students best mental energy will go toward test prep, while school work will be deprioritized. If one cannot be completed under the time constraints given, school work will be incomplete, not test prep.

Second, a student can take different classes. It is known in high schools which coursework is more rigorous than others. While AP Chemistry and AP Environmental science will both serve as an advanced –placement science course, AP Chemistry is the far more rigorous and time consuming.

Finally, a student can spread more challenging courses over summer school or community colleges. By taking a difficult course over the summer or at community college, a student can spread out more difficult courses, thus allowing more time for test prep throughout the year.

Hopefully this analysis has illustrated that under liberal and conservative time estimates, and under all potential views that colleges would use to evaluate a candidate's rank, a 7th semester senior's time is best spent preparing first for the SAT or ACT, and then for his high-school classes.